

**THE KAVERY ENGINEERING COLLEGE***(An Autonomous Institution)***B.E/B.TECH DEGREE END SEMESTER THEORY EXAMINATIONS, APRIL-MAY 2025****Fourth Semester****Artificial Intelligence and Data Science****MA3391 – Probability and Statistics****Statistical tables can be permitted****Regulations - 2021****Duration : 3 Hrs****Maximum : 100 Marks****PART – A (ANSWER ALL QUESTIONS) (Marks 10\*2=20)**

| Q.No | Questions                                                                                                | BLT | CO | Marks |
|------|----------------------------------------------------------------------------------------------------------|-----|----|-------|
| 1.   | Verify $f(x) = 6x(1-x), 0 \leq x \leq 1$ is a probability density function.                              | L2  | 1  | 2     |
| 2.   | State Memory less property of Geometric distribution.                                                    | L1  | 1  | 2     |
| 3.   | Find the regression co-efficient $b_{YX}$ , given $r = 0.394, \sigma_x = 3.741$ and $\sigma_y = 6.308$ . | L2  | 2  | 2     |
| 4.   | State central limit theorem.                                                                             | L1  | 2  | 2     |
| 5.   | Define estimator.                                                                                        | L2  | 3  | 2     |
| 6.   | Define type – I and Type – II error.                                                                     | L2  | 3  | 2     |
| 7.   | State the basic assumptions made in non-parametric tests.                                                | L1  | 4  | 2     |
| 8.   | Can we apply the Kolmogorov test for discrete distribution in non-parametric tests? If so justify.       | L2  | 4  | 2     |
| 9.   | List the control charts for attributes.                                                                  | L1  | 5  | 2     |
| 10.  | Write the lower and upper limits of c chart.                                                             | L1  | 5  | 2     |

**PART – B (ANSWER ALL QUESTIONS) (Marks 5\*16 = 80)**

| Q.No | Questions                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  | BLT | CO | Marks |    |                |                 |                    |   |   |      |   |   |    |    |    |                |                 |                    |    |   |   |
|------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----|----|-------|----|----------------|-----------------|--------------------|---|---|------|---|---|----|----|----|----------------|-----------------|--------------------|----|---|---|
| 11.  | <p>a i Three urns are there containing white and black balls; first urn has 3 white and 2 black balls, second urn has 2 white and 3 black balls and third urn has 4 white and 1 black balls. Without any biasing one urn is chosen from that one ball is chosen randomly which was white. What is probability that it came from the third urn?</p> <p>i The probability distribution</p> <table><tr><td>X</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td></tr><tr><td>P(X)</td><td>0</td><td>K</td><td>2K</td><td>2K</td><td>3K</td><td>K<sup>2</sup></td><td>2K<sup>2</sup></td><td>7K<sup>2</sup>+K</td></tr></table> <p>Find (i) the value of K (ii) <math>P(X &gt; 3)</math> (iii) <math>P(2 \leq X \leq 4)</math>.</p> <p>Or</p> | X   | 0  | 1     | 2  | 3              | 4               | 5                  | 6 | 7 | P(X) | 0 | K | 2K | 2K | 3K | K <sup>2</sup> | 2K <sup>2</sup> | 7K <sup>2</sup> +K | L2 | 1 | 8 |
| X    | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | 1   | 2  | 3     | 4  | 5              | 6               | 7                  |   |   |      |   |   |    |    |    |                |                 |                    |    |   |   |
| P(X) | 0                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          | K   | 2K | 2K    | 3K | K <sup>2</sup> | 2K <sup>2</sup> | 7K <sup>2</sup> +K |   |   |      |   |   |    |    |    |                |                 |                    |    |   |   |
| b i  | Derive Moment generating function of Binomial distribution. Hence find its mean and variance.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              | L2  | 1  | 8     |    |                |                 |                    |   |   |      |   |   |    |    |    |                |                 |                    |    |   |   |

- i The Saving bank account of a customer showed an average balance of Rs.150 and S.D of Rs.50 assuming the account balances are normally distributed, Find the % of account
- i
- Over Rs.200.
  - Between Rs.120 and Rs.170.
  - Less than 75.

L3 1 8

12. a The joint probability mass function of  $(X,Y)$  is given by  $P(X,Y) = K(x+2y)$ ,  $x=0,1,2$ ;  $y=0,1,2$ . Find the marginal and conditional probability distribution of  $x$  and  $y$ . Also find the probability distribution of  $X+Y$ .

L3 2 16

**Or**

- b The joint pdf of the two dimensional random variable  $(x, y)$  is
- $$f(x,y) = \begin{cases} 2-x-y, & 0 < x < 1 \quad 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$
- Find the covariance between  $X$  and  $Y$  and the correlation coefficient between  $X$  and  $Y$ .

L3 2 16

13. a i If  $x_1, x_2, \dots, x_n$  be a random variable with mean  $\mu$  and variance  $\sigma^2$ . Then prove that  $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$  is an unbiased estimator of  $\sigma^2$ .

L2 3 8

- i In a test given two group of students and the marks obtained are as follows:

L3 3 8

|              |    |    |    |    |    |    |    |    |    |
|--------------|----|----|----|----|----|----|----|----|----|
| First group  | 18 | 20 | 36 | 50 | 49 | 36 | 34 | 49 | 61 |
| Second group | 29 | 28 | 26 | 25 | 30 | 44 | 46 |    |    |

Construct the 95% confidence interval on the mean marks secured by the two groups.

**Or**

- b i For random sampling from Poisson distribution with parameter  $\lambda$ , obtain the maximum likelihood estimator of  $\lambda$ .

L2 3 8

- i Let  $x_1, x_2, \dots, x_n$  be a random sample from uniform distribution with density function  $f(x,a,b) = \frac{1}{b-a}$  where  $a < x < b$ . Find the estimators of  $a$  and  $b$  by the method of moments.

L2 3 8

14. a i The following are the average weekly loses of worker hours due to accidents in 10 industrial plants before and after a certain safety programme was conducted.

L3 4 8

|        |    |    |    |     |    |    |    |    |    |    |
|--------|----|----|----|-----|----|----|----|----|----|----|
| Before | 45 | 73 | 46 | 124 | 33 | 57 | 83 | 34 | 26 | 17 |
| After  | 36 | 60 | 44 | 119 | 35 | 51 | 77 | 29 | 24 | 11 |

Use the sign test at  $\alpha = 0.05$  to test if the safety programme is effective.

- i An experiment is designed to compare three methods for preventing corrosion yielded the following maximum depth of pieces of wire subject to a respective treatments. L3 4 8

| Method A | Method B | Method C |
|----------|----------|----------|
| 77       | 60       | 49       |
| 54       | 41       | 52       |
| 57       | 59       | 69       |
| 74       | 65       | 47       |
| 71       | 62       | 56       |
| 66       | 64       |          |
|          | 52       |          |

Use 5% level of significance to test the null hypothesis that three samples come from the identical population. Conduct H Test to compare the three methods.

**Or**

- b i Test whether the following two samples have been drawn from the same population, using Rank sum test. L3 4 8

|           |     |     |     |     |     |     |     |     |    |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|----|
| Sample I  | 134 | 146 | 104 | 119 | 124 | 161 | 107 | 113 | 94 |
| Sample II | 70  | 101 | 118 | 85  | 107 | 132 | 94  | 97  |    |

- i Suppose that in a study of sedimentary rocks, the following diameters (in millimeters) were obtained for two kinds of sand. L3 4 8

|      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
| S I  | 0.63 | 0.47 | 0.17 | 1.36 | 0.35 | 0.51 | 0.49 | 0.45 | 0.18 | 0.84 | 0.43 | 0.32 | 0.12 | 0.40 | 0.20 |
| S II | 1.13 | 1.01 | 0.54 | 0.48 | 0.96 | 0.89 | 0.26 | 1.07 | 0.39 | 1.11 | 0.88 | 0.58 | 0.92 | 0.53 |      |

Use the U test at 0.01 level of significance to test the null hypothesis that the two samples come from identical populations against the alternative hypothesis that the population are not identical.

15. a Given below are the value of sample mean  $\bar{x}$  and sample range R for 10 samples, each of size 5. Draw the appropriate mean and range charts and comment or the state of control of the process is either out of control or under control. L3 5 16

|         |    |    |    |    |    |    |    |    |    |    |
|---------|----|----|----|----|----|----|----|----|----|----|
| Samples | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
| Mean    | 43 | 49 | 37 | 44 | 45 | 37 | 51 | 46 | 43 | 47 |
| Range   | 5  | 6  | 5  | 7  | 7  | 4  | 8  | 6  | 4  | 6  |

**Or**

- b 10 samples each of size 50 were inspected and the number of defectives in the inspection were 2,1,1,2,3,5,1,2,3. Construct a p- L3 5 16

chart and np-chart for each sample. Assuming special causes for out of control point, find the revised control limits.